Disturbance Prediction-Based Adaptive Event-Triggered Model Predictive Control for Perturbed Nonlinear Systems

Pengfei Li, Yu Kang, Tao Wang, and Yun-Bo Zhao, *Senior Member, IEEE*

***Abstract*—A disturbance prediction-based adaptive event-triggered model predictive control scheme is proposed for nonlinear systems in the presence of slowly varying disturbance. The optimal control problem in the model predictive control scheme is formulated by taking advantage of a proposed central path-based disturbance prediction approach, and the event-triggered mechanism is designed to be adaptive to the triggering interval. As a result, the proposed scheme improves the state prediction precision and, hence, reduces greatly the triggering frequency. Furthermore, for input-affine nonlinear systems, the disturbance separation and compensation techniques are developed to further enlarge the triggering interval. The theoretical analysis of the algorithm feasibility and closed-loop stability, as well as numerical evaluations of the effectiveness of the proposed schemes, is also given.**

***Index Terms*—Disturbance prediction, event-triggered control, model predictive control (MPC), nonlinear system.**

I. Introduction

As a powerful control technique to handle nonlinear dynamics and system constraints, nonlinear model predictive control (MPC) has been playing a vital role both in industry and academia for decades [1]. In a typical nonlinear MPC algorithm, the solution to the underline finite-time optimal control problem (OCP) has to be repeatedly sought for at every time step, occupying a great amount of computing resources. This computational challenge for nonlinear MPC can be even more severe in such scenarios, e.g., large-scale systems where the total amount of the computing resources can be huge or systems with limited and extremely valuable computing resource as in miniature robots. For such scenarios, the reduction of the computing resource usages can be meaningful.

A promising approach toward this challenge is to reduce the computational frequency, i.e., the underline OCP is activated only if certain triggering condition is violated, hence the event-triggered MPC (ETMPC). One may observe that such benefits depend essentially on the triggering conditions, which becomes one of the core design issues of ETMPC.

Manuscript received 23 August 2021; revised 24 January 2022; accepted 20 April 2022. Date of publication 25 April 2022; date of current version 29 March 2023. This work was supported in part by the National Key Research and Development Program of China under Grant 2018AAA0100801, in part by the National Natural Science Foundation of China under Grant 62103394, Grant 62173317, and Grant 62033012, and in part by the Project funded by China Postdoctoral Science Foundation under Grant 2020M682036. Recommended by Associate Editor T. Faulwasser. *(Corresponding author: Yu Kang.)*

Pengfei Li, Tao Wang, and Yun-Bo Zhao are with the Department of Automation, University of Science and Technology of China, Hefei 230027, China (e-mail: puffylee@ustc.edu.cn; wanghtao@mail.ustc.edu.cn; ybzhao@ieee.org).

Yu Kang is with the Department of Automation, University of Science and Technology of China, Hefei 230027, China, and also with the Institute of Artificial Intelligence, Hefei Comprehensive National Science Center, Hefei 230088, China (e-mail: kangduyu@ustc.edu.cn).

Color versions of one or more figures in this article are available at https://doi.org/10.1109/TAC.2022.3169905.

Digital Object Identifier 10.1109/TAC.2022.3169905

The above core issue has been considered from various aspects [2]–[11]. Summarizing from these works one may find that existing triggering conditions are mainly derived either from system stability or recursive feasibility. For the former, the triggering conditions are designed by ensuring the decrement of the Lyapunov function at two successive time steps [10] or two successive triggering instants [11]. For the latter, the triggering conditions are designed by bounding the error between the actual state and the predicted one to guarantee the recursive feasibility, see, e.g., [2]–[5] for continuous-time nonlinear systems and [6]–[8] for discrete-time ones. However, most stability or feasibility conditions are only sufficient, hence the resulting triggering conditions can be naturally conservative, and consequently, these conservative triggering conditions may lead to unnecessarily high triggering frequency. To reduce the triggering frequency, the classic methodology is either to use a shrinking prediction horizon [3] or the dynamic event-triggering condition [10], whose main idea is to increase the triggering threshold.

Besides the aforementioned progress in designing a less conservative triggering condition, improving the state prediction precision, inspired by the triggering conditions for recursive feasibility in [2]–[8], is another effective way to lower the triggering frequency. The core challenge here is to reduce the state prediction error induced by the disturbance. Existing literature indicates two possible paths. The first path, usually applied to input-affine nonlinear systems, tries to suppress disturbance by using some disturbance rejection techniques, e.g., integral sliding-mode control [12] and disturbance observer with feedforward compensation [13]. The second path tries to predict the future disturbance and generate a tailored control sequence by using the predicted disturbance in formulating MPC [14], [15].

In this article, we first try to improve the state prediction precision for general nonlinear systems by predicting disturbances, based on which we improve the precision for input-affine systems by further suppressing disturbances. Compared with the conventional ETMPC [2] and periodic disturbance prediction-based MPC [14], [15], new challenges arise

1) First, how to predict the disturbance evolution over the prediction horizon such that the disturbance prediction error can be explicitly analyzed?

2) Second, how to exploit the predicted disturbance and set the constraint conditions in formulating the OCP such that the recursive feasibility can be easily guaranteed?

3) Third, how to design the triggering condition that is capable of reaping the benefit brought by the predictive state sequence with high precision?

The above-mentioned three challenges are solved by the proposed disturbance prediction-based ETMPC scheme in this article. Moreover, a refined ETMPC scheme is proposed for the input-affine nonlinear systems by using the disturbance separation method and the feedforward compensation for matched disturbance. The main contributions are listed as follows.

1) A central path-based disturbance prediction method is proposed and the corresponding prediction error is analyzed.

2) A novel OCP is presented, where the disturbance-related predicted model improves the state prediction precision, and the new constraint tightening scheme ensures the robustness.
3) Two less conservative triggering conditions are proposed, which rely on the prediction errors of the state and the disturbance, and are adaptive to the triggering interval.

Notations: For a vector $y \in \mathbb{R}^n$, let $y^{[i]}$ be the ith element of y. Let y^T, $\|y\|_2$, and $\|y\|_P$ denote the transpose, the Euclidean norm, and the P-weighted norm, respectively. For a matrix $A \in \mathbb{R}^{n\times n}$, $\|A\|$ is its spectral norm and $\bar{\lambda}(A)$ is its maximum eigenvalue. Given two nonempty sets $\mathbb{X}$ and $\mathbb{Y}$, the Minkowski set addition and Pontryagin set difference are defined by $\mathbb{X} \oplus \mathbb{Y} \triangleq \{x+y : x \in \mathbb{X}, y \in \mathbb{Y}\}$ and $\mathbb{X} \ominus \mathbb{Y} \triangleq \{x : \{x\} \oplus \mathbb{Y} \subset \mathbb{X}\}$, respectively. We adopt the conventions $\sum_{s=0}^{-1} a_s = 0$ and $\prod_{s=0}^{-1} a_s = 1$ for any $a_s \in \mathbb{R}$.

II. Disturbance Prediction-Based ETMPC for General Nonlinear Systems

In this section, we provide a comprehensive description of the disturbance prediction-based ETMPC scheme for general nonlinear system, and then discuss the recursive feasibility and stability.

A. System Description

Consider the following general discrete-time nonlinear system with additive disturbance:

$$x(k+1) = f(x(k), u(k)) + w(k) \tag{1}$$

where $x \in \mathbb{X} \subseteq \mathbb{R}^n$ is the system state, $u \in \mathbb{U} \subseteq \mathbb{R}^m$ is the control input, and $w \in \mathbb{W} \subseteq \mathbb{R}^n$ is the disturbance. Sets $\mathbb{X}$, $\mathbb{U}$, and $\mathbb{W}$ are all compact and contain the origin in their interior.

The following assumption involving the system model and the disturbance is necessary and quite mild.

Assumption 1:
1) The function $f(x,u)$ with $f(0,0)=0$ satisfies, for all $x, y \in \mathbb{X}$
$$\|f(x,u) - f(y,u)\|_P \le L_P \|x-y\|_P \;\forall u \in \mathbb{U} \tag{2}$$
where L_P is the Lipschitz constant and P is a weighted matrix.
2) For the disturbance $w = [w^{[1]}, \ldots, w^{[n]}]^T$, there exists a vector $\eta = [\eta^{[1]}, \ldots, \eta^{[n]}]^T$ such that
$$|w^{[i]}| \le \eta^{[i]},\; i = 1, \ldots, n \tag{3}$$
where $\eta^{[i]} > 0$ represents the upper bound of $w^{[i]}$.
3) There exists a vector $\delta = [\delta^{[1]}, \ldots, \delta^{[n]}]^T$ such that
$$|w^{[i]}(k+1) - w^{[i]}(k)| \le \delta^{[i]} < \eta^{[i]},\; i = 1, \ldots, n \tag{4}$$
where $\delta^{[i]} > 0$ represents the maximum change rate of $w^{[i]}$.

Remark 1: The condition 3) indicates that the disturbance is slowly varying, which makes the disturbance easier to be predicted. This assumption has also been employed in [15], where some practical examples, e.g., the road profile in vehicle suspension systems and the solar energy in power systems, are given to show its validity.

B. ETMPC Algorithm Design

1) Central Path-Based Disturbance Prediction: Since the disturbance does not change dramatically, the previous disturbance can be used to predict its current and future value. The disturbance at time $k-1$ can be written as

$$w(k-1) = x(k) - f(x(k-1), u(k-1)). \tag{5}$$

Based on $w(k-1)$, the $(N+1)$-step predictive disturbance sequence $\hat{\mathbf{w}}_{0:N}(k) = \{\hat{w}(k|k), \ldots, \hat{w}(k+N|k)\}$ is designed as follows:

$$\hat{w}(k+j-1|k) = [\hat{w}^{[1]}(k+j-1|k), \ldots, \hat{w}^{[n]}(k+j-1|k)]^T$$

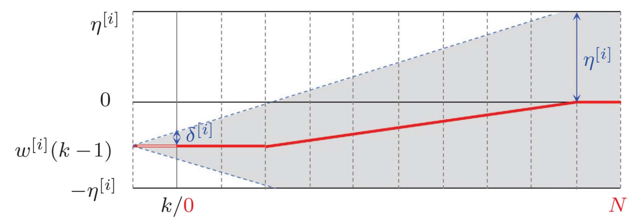

Fig. 1. Prediction of the disturbance. The actual disturbance appears in the shaded area and the red line represents the disturbance prediction $\hat{\mathbf{w}}_{0:N}(k)$ over the prediction horizon N.

$$\hat{w}^{[i]}(k+j-1|k) = \frac{1}{2}\left(\max\{w^{[i]}(k-1) - j\delta^{[i]}, -\eta^{[i]}\} + \min\{w^{[i]}(k-1) + j\delta^{[i]}, \eta^{[i]}\}\right),\; i = 1, \ldots, n \tag{6}$$

where $j = 0, 1, \ldots, N+1$ and $\hat{w}^{[i]}(k-1|k) = w^{[i]}(k-1)$. The schematic of the disturbance prediction is illustrated in Fig. 1, where the red line represents the predictive disturbance sequence $\hat{\mathbf{w}}^{[i]}_{0:N}(k)$ and the actual future disturbance can be in the shaded area. Observe that the red line is the central path of the shaded area, thus the name of "central path-based disturbance prediction."

The following lemma bounds the disturbance prediction error, which is useful in the design and analysis of ETMPC.

Lemma 1: Suppose that the disturbance satisfies conditions 2) and 3) in Assumption 1. Then for all $j = 0, 1, \ldots, N$,
1) the disturbance prediction error satisfies
$$\|w(k+j-1) - \hat{w}(k+j-1|k)\|_P \le \sqrt{\bar{\lambda}(P)}\|e(j)\|_2 \tag{7}$$
where $e(j) = [\min\{j\delta^{[1]}, \eta^{[1]}\}, \ldots, \min\{j\delta^{[n]}, \eta^{[n]}\}]^T$;
2) the difference between $\hat{w}(k+j-1|k)$ and $\hat{w}(k+j-1|k')$, $k - k' = \Delta$, $\Delta > 0$, meets
$$\|\hat{w}(k+j-1|k) - \hat{w}(k+j-1|k')\|_P \le \sqrt{\bar{\lambda}(P)}\|e(\Delta)\|_2. \tag{8}$$

Proof: See Appendix A. ■

For the ease of presentation, we define two important parameters $\bar{\eta} \triangleq \|\eta\|_2$ and $\bar{\delta} \triangleq \|\delta\|_2$. From the definition of $e(j)$, it holds that $\|e(j)\|_2 \le j\bar{\delta}$ and $\|e(j)\|_2 \le \bar{\eta}$.

2) OCP Formulation: To exploit the predictive disturbance sequence $\hat{\mathbf{w}}^{[i]}_{0:N}(k)$, in our work, the following predicted model will be used to generate the future states:

$$\hat{x}(k+1) = f(\hat{x}(k), u(k)) + \hat{w}(k). \tag{9}$$

In contrast to the nominal model that neglects the disturbance, the model in (9) considers the predicted disturbance and has the potential to generate a more precise predictive state sequence.

At time k_p (the $(p+1)$th triggering instant), suppose that the N-step predictive control sequence $\mathbf{u}(k_p) = \{u(k_p|k_p), \ldots, u(k_p+N-1|k_p)\}$ is obtained, then the corresponding state sequence $\hat{\mathbf{x}}(k_p) = \{\hat{x}(k_p+1|k_p), \ldots, \hat{x}(k_p+N|k_p)\}$ can be computed by

$$\hat{x}(k_p+j+1|k_p) = f(\hat{x}(k_p+j|k_p), u(k_p+j|k_p)) + \hat{w}(k_p+j|k_p)$$

where $\hat{x}(k_p|k_p) = x(k_p)$ and $j = 0, \ldots, N-1$.

The actual control input is given by $u(k) = u(k|k_p)$, $k_p \le k < k_{p+1}$. That is, the elements of $\mathbf{u}(k_p)$ will be applied to the plant in turn until the next triggering instant k_{p+1}.

Next, we analyze the state prediction error, which is important for the following design.

Lemma 2: Let $x(k_p+j)$ and $\hat{x}(k_p+j|k_p)$ be the actual state and the predicted one generated by systems (1) and (9) under the same inputs from $\mathbf{u}(k_p)$ and initial state $x(k_p)$, respectively, and let $\hat{x}(k_{p+1}+j|k_{p+1})$ be the predicted state evolving from the initial state $x(k_{p+1})$ and the proper inputs of $\mathbf{u}(k_p)$. Then

1) the state prediction error satisfies

$$\|x(k_p+j)-\hat{x}(k_p+j|k_p)\|_P/\sqrt{\bar{\lambda}(P)} \leq \sum_{s=0}^{j-1} L_P^s \|e(j-s)\|_2 \tag{10}$$

where $e(j)$ is defined in Lemma 1;

2) the error between the two predicted states satisfies

$$\begin{aligned}&\|\hat{x}(k_{p+1}+j|k_{p+1})-\hat{x}(k_{p+1}+j|k_p)\|_P/\sqrt{\bar{\lambda}(P)}\\ &\leq \sum_{s=0}^{\Delta_p-1} L_P^{s+j}\|e(\Delta_p-s)\|_2+\sum_{s=0}^{j-1} L_P^s\|e(\Delta_p)\|_2\end{aligned} \tag{11}$$

where $k_{p+1}-k_p=\Delta_p$ and $j\geq 0$.

Due to the limited space, the proof is omitted here.

From (10), one observes that the state prediction error, compared with those in [7], [9], and [16], has been reduced with the aid of the predicted disturbance. Indeed, such reduction can still be significant even for large disturbance as long as the disturbance changes slowly.

Based on the above lemma, the tightened constraint set $\mathbb{X}(j), j=0,\ldots,N-1$ can be designed as

$$\mathbb{X}(j)=\mathbb{X}\ominus\left\{b:\|b\|_P\leq\sum_{s=0}^{j-1}\sqrt{\bar{\lambda}(P)}(j-s)L_P^s\bar{\delta}\right\}. \tag{12}$$

With the above preparations, the OCP is described as follows:

$$\min_{\mathbf{u}(k_p)} \; J_N(x(k_p),\mathbf{u}(k_p)) \tag{13a}$$

$$\text{s.t. } \hat{x}(k_p+j+1|k_p)=f(\hat{x}(k_p+j|k_p),u(k_p+j|k_p)) + \hat{w}(k_p+j|k_p) \tag{13b}$$

$$\hat{x}(k_p|k_p)=x(k_p) \tag{13c}$$

$$\hat{x}(k_p+j|k_p)\in\mathbb{X}(j) \tag{13d}$$

$$u(k_p+j|k_p)\in\mathbb{U},\; j=0,\ldots,N-1 \tag{13e}$$

$$\hat{x}(k_p+N|k_p)\in\mathbb{X}_f \tag{13f}$$

where $J_N(x(k_p),\mathbf{u}(k_p))=\sum_{j=0}^{N-1} l(\hat{x}(k_p+j|k_p),u(k_p+j|k_p))+F(\hat{x}(k_p+N|k_p))$ is the MPC cost, N is the prediction horizon, $l(x,u)=\|x\|_Q^2+\|u\|_R^2$ is the stage cost, $F(x)=\|x\|_P^2$ is the terminal cost, P,Q,R are three positive definite matrices, and $\mathbb{X}_f=\{x|\|x\|_P<\varepsilon_f\}$ is the terminal constraint set. Note that P,Q,R, and $\mathbb{X}_f$ should satisfy some fairly standard conditions in the following assumption, which can also be found in [7], [18], and [19].

Assumption 2: There exist an auxiliary set $\mathbb{X}_a$ with the form of $\mathbb{X}_a=\{x|\|x\|_P\leq\varepsilon_a\}$ and an auxiliary control law $\kappa(x):\mathbb{X}_f\to\mathbb{U}$, such that

1) $\mathbb{X}_a\subseteq\mathbb{X}_f\subseteq\mathbb{X}(N)$;
2) $f(x,\kappa(x))\in\mathbb{X}_a, \forall x\in\mathbb{X}_f$;
3) $F(f(x,\kappa(x))+w)-F(x)\leq -l(x,\kappa(x))+\rho(\bar{\eta}), \quad \forall x\in\mathbb{X}_f$, where ρ is a $\mathcal{K}_\infty$-function.

3) Triggering Condition and Recursive Feasibility:

Assume that OCP (13) is solved at each triggering time k_p. Let $\mathbf{u}^*(k_p)=\{u^*(k_p|k_p),\ldots,u^*(k_p+N-1|k_p)\}$ represent the N-step optimal control sequence and the corresponding state sequence is denoted by $\hat{\mathbf{x}}(k_p)=\{\hat{x}(k_p|k_p),\ldots,\hat{x}(k_p+N|k_p)\}$, where $\hat{x}(k_p|k_p)=x(k_p)$. Let k_{p+1} be the next triggering instant. Then we construct the candidate N-step control sequence $\bar{\mathbf{u}}(k_{p+1})$ as follows:

$$\begin{aligned}&\bar{u}(k_{p+1}+j|k_{p+1})\\ &=\begin{cases} u^*(k_p+j+\Delta_p|k_p), & 0\leq j\leq N-\Delta_p-1\\ \kappa(\hat{x}(k_p+N|k_p)), & j=N-\Delta_p\\ \kappa(\bar{x}(k_{p+1}+j|k_{p+1})), & N-\Delta_p+1\leq j\leq N-1\end{cases}\end{aligned} \tag{14}$$

where $\bar{x}(k_{p+1}+j+1|k_{p+1})=f(\bar{x}(k_{p+1}+j|k_{p+1}),\bar{u}(k_{p+1}+j|k_{p+1}))+\hat{w}(k_{p+1}+j|k_{p+1})$, $j=0,\ldots,N$, and $\bar{x}(k_{p+1}|k_{p+1})=x(k_{p+1})$.

Theorem 1: For the system in (1) with Assumption 1, its OCP (13) is recursively feasible if the following inequalities hold:

$$\bar{\delta}\leq\frac{(\varepsilon_f-\varepsilon_a-\sqrt{\bar{\lambda}(P)}\bar{\eta})(L_P-1)}{(L_P^{N+1}-1)} \tag{15a}$$

$$\bar{\eta}\leq(\varepsilon_f-\varepsilon_a)/\sqrt{\bar{\lambda}(P)} \tag{15b}$$

and the event-triggered mechanism is designed as

$$\begin{aligned}k_{p+1}&=\min\{k_p+N,r_{p+1}\}\\ r_{p+1}&=\inf_k\left\{k\middle|\sum_{s=0}^{N-(k-k_p)} L_P^{-s}\|\hat{w}(k+s|k)-\hat{w}(k+s|k_p)\|_P\right.\\ &\left.+L_P\|x(k)-\hat{x}(k|k_p)\|_P>\frac{\varepsilon_f-\varepsilon_a-\sqrt{\bar{\lambda}(P)}\bar{\eta}}{L_P^{N-(k-k_p)}}\right\}.\end{aligned} \tag{16}$$

Proof: To show the feasibility of OCP (13) at k_{p+1}, the N-step control sequence $\bar{\mathbf{u}}(k_{p+1})$ is constructed as (14). In what follows, we proceed to prove from four aspects that $\bar{\mathbf{u}}(k_{p+1})$ is feasible.

1) To prove $\bar{x}(k_{p+1}+j|k_{p+1})\in\mathbb{X}(j), \quad \forall j=1,\ldots,N-\Delta_p$, we note that $\|\hat{x}(k_p+j+\Delta_p|k_p)+\bar{x}(k_{p+1}+j|k_{p+1})-\hat{x}(k_p+j+\Delta_p|k_p)\|_P\leq\|\hat{x}(k_p+j+\Delta_p|k_p)\|_P+\|\bar{x}(k_{p+1}+j|k_{p+1})-\hat{x}(k_p+j+\Delta_p|k_p)\|_P$. Recalling that $\|e(j)\|_2\leq j\bar{\delta}$ and $\hat{x}(k_p+j+\Delta_p|k_p)\in\mathbb{X}(j+\Delta_p)$, we obtain $\bar{x}(k_{p+1}+j|k_{p+1})\in\mathbb{X}(j+\Delta_p)\oplus(\sum_{s=j}^{\Delta_p+j-1}\sqrt{\bar{\lambda}(P)}(\Delta_p+j-s)L_P^s\bar{\delta}+\sum_{s=0}^{j-1}\sqrt{\bar{\lambda}(P)}\Delta_p L_P^s\bar{\delta})\in\mathbb{X}(j)$.
2) To prove $\bar{x}(k_{p+1}+j|k_{p+1})\in\mathbb{X}(j), \forall j=N-\Delta_p+1,\ldots,N$, we show that $\bar{x}(k_p+N+1|k_{p+1})\in\mathbb{X}_f$. Note that $\|\bar{x}(k_p+N+1|k_{p+1})\|_P\leq\|f(\hat{x}(k_p+N|k_p),\kappa(\hat{x}(k_p+N|k_p)))+\hat{w}(k_p+N|k_p)\|_P+\sum_{s=0}^{N-\Delta_p}L_P^{N-\Delta_p-s}\|\hat{w}(k_{p+1}+s|k_{p+1})-\hat{w}(k_{p+1}+s|k_p)\|_P+L_P^{N-\Delta_p+1}\|x(k_{p+1})-\hat{x}(k_{p+1}|k_p)\|_P$. According to the triggering condition (16) and the property 2) in Assumption 2, we have $\|\bar{x}(k_p+N+1|k_{p+1})\|_P\leq\varepsilon_a+\sqrt{\bar{\lambda}(P)}\bar{\eta}+\varepsilon_f-\varepsilon_a-\sqrt{\bar{\lambda}(P)}\bar{\eta}\leq\varepsilon_f$, i.e., $\bar{x}(k_p+N+1|k_{p+1})\in\mathbb{X}_f$. On the basis of the property 2) in Assumption 2 and (15b), we can claim that $\mathbb{X}_f$ is a robust positively invariant set. As a result, the future states will stay in $\mathbb{X}_f$, i.e., $\bar{x}(k_{p+1}+j|k_{p+1})\in\mathbb{X}_f\subseteq\mathbb{X}(j)$, $\forall j=N-\Delta_p+1,\ldots,N$.
3) Note that $\bar{x}(k_{p+1}+N|k_{p+1})\in\mathbb{X}_f$ has been verified in 2).
4) Since $\bar{x}(k_{p+1}+j|k_{p+1})\in\mathbb{X}_f$, $\forall j=N-\Delta_p+1,\ldots,N$, the control constraint satisfaction, i.e., $u(k_{p+1}+j|k_{p+1})\in\mathbb{U}$, $j=0,\ldots,N-1$, can be directly verified based on the definition of κ in Assumption 2 and (15b).

Remark 2: The triggering condition (16) is distinct from conventional ones in two aspects. First, unlike conventional triggering conditions with constant threshold, e.g., [6], [7], the triggering threshold $(\varepsilon_f-\varepsilon_a-\sqrt{\bar{\lambda}(P)}\bar{\eta})/L_P^{N-(k-k_p)}$ in (16) is time-varying and increasing with respect to the triggering interval $k-k_p$. Consequently, the amount of triggering can be reduced. Second, the disturbance prediction error is added in the triggering condition, which seems to make the condition more likely to be triggered. But if the disturbance changes slowly, the effect of the disturbance prediction error is trivial and the

Algorithm 1: Disturbance prediction-based ETMPC.

1: Measure the current state $x(k)$;
2: Generate $\hat{\mathbf{w}}_{0:N}(k)$ according to (6);
3: **if** condition (16) is triggered **then**
4: Update the triggering instant $p \leftarrow p+1$, $k_p = k$;
5: Solve OCP (13) to generate $\hat{\mathbf{x}}(k_p)$ and $\mathbf{u}^*(k_p)$;
6: **end if**
7: Apply the control input $u(k|k_p)$ to the plant;
8: Update the time instant $k \leftarrow k+1$, and go to 1.

state prediction error is also small. Therefore, the proposed triggering condition is suitable for slowly varying disturbance.

Algorithm 1 summarizes the presented ETMPC scheme in this article.

Remark 3: Algorithm 1 is distinct from the periodic disturbance prediction-based MPC algorithm and the conventional ETMPC algorithm in two aspects. First, compared with the disturbance prediction method in [15] where the disturbance prediction sequence is arbitrarily selected from all possible disturbance realizations, the method in (6) has a lower disturbance prediction error. Second, compared with the OCP and the event-triggering condition in [2] and [7], the OCP in (13) explicitly considers the disturbance predicted error while ensuring the recursive feasibility, and the triggering condition (16) relies on the prediction errors of the state and the disturbance, lowering the triggering frequency.

C. Stability Analysis

In what follows, we carry out the stability analysis for the system with the designed ETMPC scheme.

Theorem 2: The system in (1) with Algorithm 1 is input-to-state stable under Assumptions 1 and 2.

Proof: The proof is divided into two steps. 1) Show that there exist $\bar{\beta} \in \mathcal{KL}$ and $\bar{\eta} \in \mathcal{K}$ such that $\|x(k_p)\| \le \bar{\beta}(\|x(k_0)\|, k_p - k_0) + \bar{\gamma}(\bar{\eta}), \forall k_p$ holds at each triggering instant by verifying that the optimal MPC cost $J_N(x(k_p), \mathbf{u}^*(k_p))$ ($J_N(x(k_p))$ in short) is an ISS Lyapunov function. 2) Show that there exist $\beta \in \mathcal{KL}$ and $\eta \in \mathcal{K}$ such that $\|x(k)\| \le \beta(\|x(k_0)\|, k - k_0) + \gamma(\bar{\eta}), \forall k$ by bounding $\|x(k)\|, k \in [k_p, k_{p+1})$. Note that the proof of step 2) can be directly followed from [20], thus it is omitted here. In the following part, we focus on the proof of step 1).

Note that there exist $\alpha_1, \alpha_2 \in \mathcal{K}$ such that $\alpha_1(\|x\|) \le J_N(x) \le \alpha_2(\|x\|)$ holds, one only needs to discuss $J_N(k_{p+1}) - J_N(k_p)$ to show that $J_N(x)$ is an ISS Lyapunov function at each triggering instant. Notice first that based on Assumption 2, we have

$$\begin{aligned} F(\hat{x}(k_p+N+1|k_p)) &\le F(\hat{x}(k_p+N|k_p)) \\ &\quad - l(\hat{x}(k_p+N|k_p), \kappa(\hat{x}(k_p+N|k_p))) + \rho(\bar{\eta}) \end{aligned} \tag{17}$$

where $\hat{x}(k_p+N+1|k_p) = f(\hat{x}(k_p+N|k_p), \kappa(\hat{x}(k_p+N|k_p))) + \hat{w}(k_p+N|k_p)$. Similarly, since $\bar{x}_{N-\Delta_p+j}(k_{p+1}) \in \mathbb{X}_f$, $j = 1, \ldots, \Delta_p$ (as proved in Theorem 1), one can also obtain that

$$\begin{aligned} F(\bar{x}(k_{p+1}+N|k_{p+1})) &\le (\Delta_p - 1)\rho(\bar{\eta}) + F(\bar{x}(k_p+N-1|k_{p+1})) \\ &\quad - \sum_{s=N-\Delta_p+1}^{N-1} l\left(\bar{x}(k_{p+1}+s|k_{p+1}), \bar{u}(k_{p+1}+s|k_{p+1})\right) \end{aligned} \tag{18}$$

where the control sequence $\bar{\mathbf{u}}(k_{p+1})$ in (14) is feasible.

According to the definition of $J_N(x)$, one obtains

$$\begin{aligned} &J_N(x(k_{p+1})) - J_N(x(k_p)) \\ &\le -\sum_{s=0}^{N-1} l(\hat{x}(k_p+s|k_p), u^*(k_p+s|k_p)) \\ &\quad - F(\hat{x}(k_p+N|k_p)) \\ &\quad + \sum_{s=0}^{N-\Delta_p} l(\bar{x}(k_{p+1}+s|k_{p+1}), \bar{u}(k_{p+1}+s|k_{p+1})) \\ &\quad + \sum_{s=N-\Delta_p+1}^{N-1} l\left(\bar{x}(k_{p+1}+s|k_{p+1}), \bar{u}(k_{p+1}+s|k_{p+1})\right) \\ &\quad + F(\bar{x}(k_{p+1}+N|k_{p+1})). \end{aligned} \tag{19}$$

Note that $\|x\|_P^2 - \|y\|_P^2 \le L_F\|x-y\|_P, \forall x, y \in \mathbb{X}_f$ and $\|x\|_Q^2 - \|y\|_Q^2 \le L_l\|x-y\|_P, \forall x, y \in \mathbb{X}$ hold with $L_F \triangleq 2\min_{x\in\mathbb{X}_f}\|x\|_P$, $L_l \triangleq 2\sqrt{\frac{\bar{\lambda}(Q)}{\underline{\lambda}(P)}}\min_{x\in\mathbb{X}}\|x\|_Q$. Then, substituting (17) and (18) into (19) and incorporating (11) yield

$$\begin{aligned} &J_N(x(k_{p+1})) - J_N(x(k_p)) \\ &\le -\|x(k_p)\|_Q^2 + \sum_{j=0}^{N-\Delta_p} \|\bar{x}(k_{p+1}+j|k_{p+1})\|_Q^2 - \|\hat{x}(k_{p+1}+j|k_p)\|_Q^2 \\ &\quad - \|\hat{x}(k_p+N+1|k_p)\|_P^2 + \|\bar{x}(k_p+N+1|k_{p+1})\|_P^2 + \Delta_p\rho(\bar{\eta}) \\ &\le -\|x(k_p)\|_Q^2 + \alpha(\bar{\eta}) \end{aligned} \tag{20}$$

where $\alpha(\bar{\eta}) \triangleq \Delta_p\rho(\bar{\eta}) + \sqrt{\bar{\lambda}(P)}(\sum_{j=0}^{N-\Delta_p} L_l\sqrt{\frac{\bar{\lambda}(Q)}{\underline{\lambda}(P)}}\frac{L_P^{j+\Delta_p}-1}{L_P-1} + L_F\frac{L_P^{N+1}-1}{L_P-1})\bar{\eta}$ is a $\mathcal{K}_\infty$ function. By [1, Lemma B.38], the above procedures complete step 1).

Following the proof line of [23, Th. 1], we can easily complete the proof of step 2), which suggests the input-to-state stability. ■

III. Input-Affine Nonlinear Case

Based on the results in Section II, this section discusses input-affine nonlinear systems. A less conservative result is yielded by using the disturbance separation and compensation technique.

A. System Description and Disturbance Separation

Consider the case where $f(x, u)$ in (1) has the following form:

$$f(x, u) = g(x) + B(x)u \tag{21}$$

where matrix $B \in \mathbb{R}^{n\times m}$ is state-dependent and $\text{rank}(B(x)) = m$. For system (1) with the above special form, we also assume that Assumption 1 holds.

It can be observed from Theorem 1 that the disturbance plays a key role in determining the triggering instant. A larger triggering interval can be obtained if some extra disturbance rejection technique is employed. First, by separating the unmatched disturbance $w_U(k)$ (i.e., $w_U(k) \notin \text{Range}(B(x)), \exists x \in \mathbb{X}$) from the matched counterpart $w_M(k)$ (i.e., $w_M(k) \in \text{Range}(B(x)), \forall x \in \mathbb{X}$), the disturbance $w(k)$ can be expressed as

$$w(k) = w_M(k) + w_U(k) \tag{22a}$$

$$w_M(k) = B(x)G(x)w(k) \tag{22b}$$

$$w_U(k) = (I - B(x)G(x))w(k) \tag{22c}$$

where $G \in \mathbb{R}^{m\times n}$ is a state-dependent matrix.

$G(x)$ plays an essential role in the above decomposition. Indeed, since the matched disturbance can be offset by feedforward compensation while the unmatched one cannot [21], we hope w_U in (22) is as small as possible. Specifically, we need to choose a proper $G(x)$ such that $\|w_U\|_P$ is minimized.

Lemma 3: The choice $G(x) = (B^T(x)PB(x))^{-1}B^T(x)P$ minimizes $\|w_U\|_P$, i.e.,

$$(B^T(x)PB(x))^{-1}B^T(x)P = \arg\min_{G(x)} \|(I - B(x)G(x))w\|_P.$$

Proof: Notice first that

$$\|(I - B(x)G(x))w\|_P = \|V(I - B(x)G(x))w\|_2$$

where V is an invertible matrix such that $V^TV = P$. Let $\phi = Vw$ and $\varphi = G(x)w$, then following the similar lines presented in [25, Prop. 2], the following optimization problem:

$$\min_{\varphi\in\mathbb{R}^m} \|\phi - VB(x)\varphi\|_2 \tag{23}$$

has optimal solution $\varphi^* = (B^T(x)V^TVB(x))^{-1}B^T(x)V^T\phi$.

Letting $G(x) = (B^T(x)PB(x))^{-1}B^T(x)P$, we have

$$\varphi = (B^T(x)PB(x))^{-1}B^T(x)Pw = \varphi^*.$$

which completes the proof.

B. ETMPC Algorithm Design

1) Computing of Control Sequence: In order to reduce the uncertainty in the predicted model, we split the control input into two parts to address the matched and unmatched disturbance separately. To be specific, the control input u has the following form:

$$u(k) = v(k) + \nu(k) \tag{24}$$

where $v(k)$ is obtained by solving an OCP and $\nu(k)$ adopts the following control law to compensate for the matched disturbance:

$$\nu(k) = -G(\hat{x}(k))\hat{w}(k). \tag{25}$$

Substituting (24) and (25) into (9) and (21), a new predicted model is formed as follows:

$$\hat{x}(k+1) = g(\hat{x}(k)) + B(\hat{x}(k))v(k) + \hat{w}_U(k). \tag{26}$$

Since x and w are both bounded, there exists a bounded set $\mathcal{V}$ such that $\nu(k) \in \mathcal{V}$. Then, if $v(k) \in \mathbb{U} \ominus \mathcal{V}$, we have $u(k) \in \mathbb{U}$.

With the above preparations, the OCP can be described as follows:

$$\min_{\mathbf{v}(k_p)} \; J_N(x(k_p), \mathbf{v}(k_p)) \tag{27a}$$

$$\text{s.t. } \hat{x}(k_p + j + 1|k_p) = g(\hat{x}(k_p + j|k_p)) + \hat{w}_U(k_p + j|k_p) + B(\hat{x}(k_p + j|k_p))v(k_p + j|k_p) \tag{27b}$$

$$\hat{x}_0(k_p) = x(k_p) \tag{27c}$$

$$\hat{x}(k_p + j|k_p) \in \mathbb{X}(j) \tag{27d}$$

$$v(k_p + j|k_p) \in \mathbb{U} \ominus \mathcal{V},\; j = 0, \ldots, N-1 \tag{27e}$$

$$\hat{x}(k_p + N|k_p) \in \mathbb{X}_f \tag{27f}$$

where $\hat{w}_U(k_p + j|k_p) = (I - B(\hat{x}(k_p + j|k_p))G(\hat{x}(k_p + j|k_p))) \times \hat{w}(k_p + j|k_p)$, $\mathbb{X}(j)$ and $J_N(x(k_p), \mathbf{v}(k_p))$ maintains the same form and parameters as those in (13). Sets $\mathbb{X}_f$ and $\mathbb{X}_a$ should be redesigned to meet Assumption 2 as the control constraint has been changed.

At each triggering instant k_p, solving the above OCP yields the N-step control sequence $\mathbf{v}(k_p) = \{v(k_p|k_p), \ldots, v(k_p + N - 1|k_p)\}$, then the corresponding state sequence is $\hat{\mathbf{x}}(k_p)$. Incorporating (24) and (25), the actual N-step control sequence $\mathbf{u}(k_p)$ with each element $u(k_p + j|k_p)$, $j = 0, \ldots, N-1$, has the following expression:

$$u(k_p + j|k_p) = v(k_p + j|k_p) - G(\hat{x}(k_p + j|k_p))\hat{w}_j(k_p + j|k_p). \tag{28}$$

2) Triggering Condition and Recursive Feasibility: Now, to establish the recursive feasibility with the designed $\mathbf{u}(k_p)$, we need to reconsider Lemma 2. Indeed, it can be easily verified that (10) and (11) still hold. In particular, for $\|\hat{x}(k_{p+1} + j|k_{p+1}) - \hat{x}(k_{p+1} + j|k_p)\|_P$ in (11), a tighter upper bound may be obtained.

Let $\Gamma(x) = I - B(x)G(x)$ and define a positive constant L_Γ such that $\|V(\Gamma(x) - \Gamma(y))\| \le L_\Gamma\|x - y\|_P$ holds for any $x, y \in \mathbb{X}$, where $P = V^TV$. Noting that $\|(\Gamma(x) - \Gamma(y))w\|_P = \|V(\Gamma(x) - \Gamma(y))w\|_2 \le \|V(\Gamma(x) - \Gamma(y))\|\|w\|_2 \le L_\Gamma\|w\|_2\|x - y\|_P$, $\forall w \in \mathbb{W}$, we then obtain

$$\begin{aligned}
&\|\hat{x}(k_{p+1} + j|k_{p+1}) - \hat{x}(k_{p+1} + j|k_p)\|_P \\
&\le \bar{L}_P\|\hat{x}(k_{p+1} + j - 1|k_{p+1}) - \hat{x}(k_{p+1} + j - 1|k_p)\|_P \\
&\quad + \|\Gamma(\hat{x}(k_{p+1} + j - 1|k_{p+1}))\hat{w}(k_{p+1} + j - 1|k_{p+1}) \\
&\qquad - \Gamma(\hat{x}(k_{p+1} + j - 1|k_p))\hat{w}(k_{p+1} + j - 1|k_p)\|_P \\
&\le \prod_{s=0}^{j-1}\left(\bar{L}_P + L_\Gamma\|\hat{w}(k_{p+1} + s|k_{p+1})\|_2\right)\|x(k_{p+1}) - \hat{x}(k_{p+1}|k_p)\|_P \\
&\quad + \sum_{s=0}^{j-1}\prod_{r=s}^{j-2}\left(\bar{L}_P + L_\Gamma\|\hat{w}(k_{p+1} + r|k_{p+1})\|_2\right)\|\Gamma(\hat{x}(k_{p+1} + s|k_p)) \\
&\quad \times (\hat{w}(k_{p+1} + s|k_{p+1}) - \hat{w}(k_{p+1} + s|k_p))\|_P
\end{aligned} \tag{29}$$

where $\bar{L}_P$ is a constant such that $\|g(x) - B(x)u - g(y) - B(y)u\|_P \le \bar{L}_P\|x - y\|, \forall u \in \mathbb{U} \ominus \mathcal{V}, \forall x, y \in \mathbb{X}$. One notices that $\bar{L}_P \le L_P$ (L_P is given in Assumption 1) as the control constraint is tightened.

Based on (29) and following the idea of establishing the recursive feasibility in Theorem 1, we directly obtain the following result.

Theorem 3: For input-affine nonlinear systems, OCP (27) is recursively feasible if the disturbance satisfies condition (15) and the triggering condition is designed as

$$\begin{aligned}
k_{p+1} &= \min\{k_p + N, r_{p+1}\} \\
r_{p+1} &= \inf_k \Bigg\{ k \Bigg| \sum_{s=0}^{N-(k-k_p)} \prod_{r=s}^{N-(k-k_p)-1} \left(\bar{L}_P + L_\Gamma\|\hat{w}(k + r|k)\|_2\right) \\
&\quad \times \|\Gamma(\hat{x}(k + s|k_p))(\hat{w}(k + s|k) - \hat{w}(k + s|k_p))\|_P \\
&\quad + \prod_{s=0}^{N-(k-k_p)} \left(\bar{L}_P + L_\Gamma\|\hat{w}(k + s|k)\|_2\right)\|x(k) - \hat{x}(k|k_p)\|_P \\
&\quad > \varepsilon_f - \varepsilon_a - \sqrt{\bar{\lambda}(P)}\bar{\eta} \Bigg\}.
\end{aligned} \tag{30}$$

Proof: Following the same arguments as in Theorem 1 and considering both Lemma 2 and (29), the recursive feasibility can then be established. ■

Remark 4: The following two special cases are noteworthy.

1) *B is a constant matrix:* In this case, we can set $L_\Gamma = 0$ and $\Gamma = I - B(B^TPB)^{-1}B^TP$, and then the key inequality in triggering condition (30) becomes

$$\sum_{s=0}^{N-(k-k_p)} \bar{L}_P^{-s}\|\Gamma \times (\hat{w}(k + s|k) - \hat{w}(s + k|k_p))\|_P + \bar{L}_P\|x(k) - \hat{x}(k|k_p)\|_P > \frac{\varepsilon_f - \varepsilon_a - \sqrt{\bar{\lambda}(P)}\bar{\eta}}{L_P^{N-(k-k_p)}} \tag{31}$$

which has similar form as (16). But compared with (16), the uncertainty reduction in the left-hand side of (31) achieved by

Algorithm 2: Disturbance prediction-based ETMPC (input-affine case).

1: Measure the current state $x(k)$;
2: Generate $\hat{\mathbf{w}}_{0:N}(k)$ according to (6);
3: **if** condition (30) is triggered **then**
4: Update the triggering instant $p \leftarrow p+1$, $k_p = k$;
5: Solve OCP (27) to obtain $\hat{\mathbf{x}}(k_p)$ and $\mathbf{v}^*(k_p)$;
6: Genarate the actual control sequence $\mathbf{u}(k_p)$ based on (28);
7: **end if**
8: Apply $u(k|k_p)$ to the plant;
9: Update the time instant $k \leftarrow k+1$, and go to 1.

TABLE I
PARAMETERS DESCRIPTION

Symbols	Value	Symbols	Value
ρ	1000 kg/m^3	C_p	0.239 kJ/kg.K
ΔH	-5×10^4 kJ/kmol	E/R	8750 K
k_0	7.2×10^{10} min^{-1}	U	54.94 kJ/min.m^2.K
F_0	0.1 m^3/min	T_0	350 K
r	0.219 m	c_0	1 kmol/m^3

compensating for the matched disturbance can lead to a larger triggering interval.

2) *w is matched disturbance:* In this case, $\Gamma(\hat{x}(k_p+j|k_p))\hat{w}(k_p+j|k_p)\equiv 0$; therefore, the disturbance term does not appear in (27b), and the key inequality, based on (29), becomes

$$\bar{L}_P^{N-(k-k_p)+1}\|x(k)-\hat{x}(k|k_p)\|_P > \varepsilon_f - \varepsilon_a \tag{32}$$

which has similar form as those in [6] and [7]. Compared with those works, the state prediction error $\|x(k)-\hat{x}(k|k_p)\|_P$ is reduced due to the disturbance compensation, resulting in a larger triggering interval. Moreover, the restriction that $\bar{\eta} < (\varepsilon_f-\varepsilon_a)/\sqrt{\bar{\lambda}(P)}$ is removed in this case and the disturbance variation rate should satisfy $\bar{\delta} \le (\varepsilon_f-\varepsilon_a)/\bar{L}_P^N$, which lowers the conservativeness of the ETMPC scheme.

For the input-affine nonlinear case, the procedures of the ETMPC scheme are shown in Algorithm 2.

IV. SIMULATION EXAMPLE

In this section, the proposed ETMPC schemes are applied to a well-stirred chemical reactor in [1] to show the effectiveness. The system model is described by

$$\frac{dc}{dt} = \frac{F_0(c_0-c)}{\pi r^2 h} - k_0 c e^{-\frac{E}{RT}} + w_1$$
$$\frac{dT}{dt} = \frac{F_0(T_0-T)}{\pi r^2 h} + \frac{-\Delta H}{\rho C_p} k_0 c e^{-\frac{E}{RT}} + \frac{2U}{r\rho C_p}(T_c - T) + w_2$$
$$\frac{dh}{dt} = \frac{F_0 - F}{\pi r^2} + w_3$$

where c is the molar concentration of species A, h is the level of the tank, T is the actor temperature, F is the outlet flowrate, and T_c is the coolant liquid temperature. w_1, w_2, and w_3 are external disturbances. The state and control input constraints are given by $0 \le c \le 1$ kmol/m^3, 280 K $\le T \le 370$ K, 0.5 m $\le h \le 0.8$ m, 280 K$\le T_c \le 370$ K, and $0 \le F \le 0.5$ m^3/min. The model parameters in nominal conditions are given in Table I. The initial condition is $c(0) = 0.38$ kmol/m^3, $T(0) = 320$ K, $h(0) = 0.6$ m, respectively. The steady-state operating conditions are set as

$$c^s = 0.5 \text{ kmol/m}^3,\;\; T^s = 350 \text{ K},\;\; h^s = 0.6637 \text{ m}$$

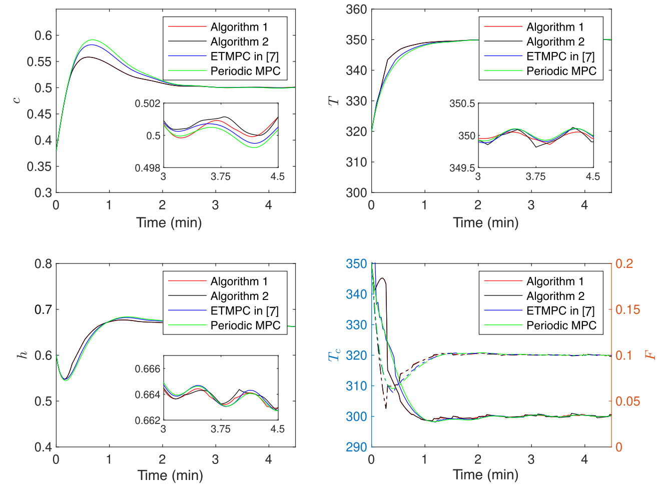

Fig. 2. State evolution and control input under the four MPC schemes. The last part of the figure illustrates the coolant liquid temperature T_c (solid) and the outlet flowrate F (dashed).

$$T_c^s = 300 \text{ K},\;\; F^s = 0.1 \text{ m}^3/\text{min}.$$

Define the state and the control input as $x = [c-c^s, T-T^s, h-h^s]^T$ and $u = [T_c - T_c^s, F-F^s]^T$, respectively, and adopt the forward-Euler discretized method with sampling interval $T_p = 0.03$ min to obtain the discrete-time system.

The parameters of the MPC schemes are set as follows. The prediction horizon is $N = 8$. Following the method proposed in [23], we set the weighted matrices as $Q = \mathrm{diag}\{1, \frac{1}{1500}, 1\}$, $R = \mathrm{diag}\{\frac{1}{1500}, 10\}$ and $P = \begin{bmatrix} 106.0759 & 1.9490 & -17.1179 \\ 1.9490 & 0.0653 & -0.1469 \\ -17.1179 & -0.1469 & 33.6757 \end{bmatrix}$. The local auxiliary controller is designed as $\kappa(x) = \begin{bmatrix} 102.4288 & -3.2468 & 8.4873 \\ -0.1769 & -0.0014 & 0.3639 \end{bmatrix} x$ to satisfy Assumption 2. Two parameters of the terminal ingredient are set to $\epsilon_f = 1.5266$ and $\epsilon_a = 1.4478$. In particular, we obtain

$$G(x) = \begin{bmatrix} 460.2751 & 15.8782 & 0 \\ 1.9179 & 0 & -5.0225 \end{bmatrix}$$

for Algorithm 2 according to Lemma 3, resulting in that set $\mathcal{V}$ fulfills $\mathcal{V} = \{[T_c, F]^T \,|\, |T_c| \le 460.2751\eta^{[1]} + 15.8782\eta^{[2]}, |F| \le 1.9179\eta^{[1]} + 5.0225\eta^{[3]}\}$.

To illustrate the effectiveness of the proposed ETMPC schemes, we compare our results with the conventional periodic MPC in [17] and the ETMPC in [7], where the key inequality in the triggering condition is $\|x(k)-\hat{x}(k|k_p)\|_2 \ge \frac{\varepsilon_f-\varepsilon_a}{\sqrt{\bar{\lambda}(P)} L_P^N}$. To ensure the recursive feasibility of these four MPC schemes, the disturbance should fulfill constraint (15) and the one reported in [17]. It is assumed that the disturbance is $w_1 = 0.0066\sin(7t-4)$, $w_2 = 0.7101\sin(8t)$, $w_3 = 0.0062\sin(9t+3)$, which means $\bar{\eta} = 0.007$ and $\bar{\delta} = 0.0018$. The simulation results are depicted in Figs. 2 and 3. As can be seen, the control objective can be achieved by all these MPC schemes without violating system constraints. In Fig. 3, Algorithms 1 and 2 are triggered 36 and 25 times, respectively, which present a significant reduction compared with the schemes in [7] (57 times) and [17] (150 times) with little sacrifice of control performance. This indicates that the adaptive triggering condition in (16) or (30) as well as the consideration of the predictive disturbance sequence in OCP (13) or (27) brings benefit in reducing the conservativeness. Moreover, since the matched disturbance has been compensated in Algorithm 2, the effect of the uncertainty is reduced in triggering condition (30), leading to the larger triggering intervals.

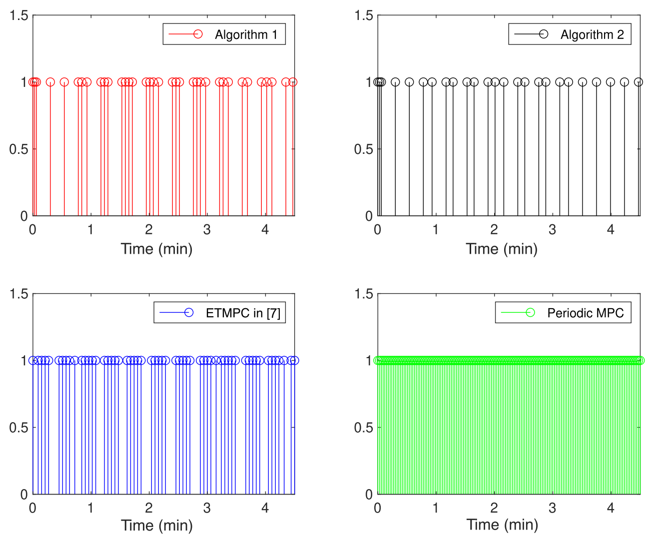

Fig. 3. Triggering time instants under the four MPC schemes.

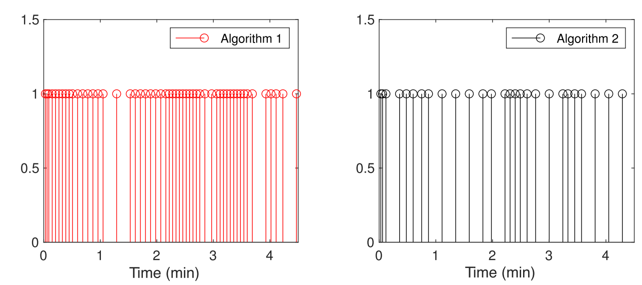

Fig. 4. Triggering time instants under Algorithms 1 and 2.

It is worth indicating that the admissible disturbance bound of our proposed ETMPC scheme can be much larger than the most existing ones if the disturbance changes slowly. Fig. 4 gives the triggering instants when the disturbance is $w_1 = 0.0314\sin(2t-1), w_2 = 1.0021\sin(4t), w_3 = 0.0521\sin(3t+1)$, i.e., $\bar{\eta} = 0.02$ and $\bar{\delta} = 0.0015$. It shows once again that the disturbance compensation technique is advantageous in reducing the triggering frequency.

V. Conclusion

Two disturbance prediction-based ETMPC schemes are proposed for general nonlinear systems and input-affine nonlinear systems, respectively. It is shown that the utilization of the predictive disturbance sequence can reduce the state prediction error, hence enlarging the triggering interval. The disturbance separation and compensation technique is also shown to be beneficial to reduce uncertainty. Numerical examples show that the triggering frequency can be significantly reduced by the proposed two ETMPC schemes. Further investigations will be focused on the applications to real-world systems.

Appendix A

First, we show that the disturbance prediction error satisfies (7). In fact, according to the definition of $\hat{w}^{[i]}(k+j|k)$, we obtain $w^{[i]}(k+j-1) - \hat{w}^{[i]}(k+j-1|k) \leq \min\{w^{[i]}(k-1)+j\delta^{[i]}, \eta^{[i]}\} - \hat{w}^{[i]}(k+j-1|k) \leq \frac{1}{2}(\min\{w^{[i]}(k-1)+j\delta^{[i]}, \eta^{[i]}\} - \max\{w^{[i]}(k-1)-j\delta^{[i]}, -\eta^{[i]}\}) \leq \frac{1}{2}(\min\{w^{[i]}(k-1)+j\delta^{[i]}, \eta^{[i]}\} + \min\{-w^{[i]}(k-1)+j\delta^{[i]}, \eta^{[i]}\}) \leq \min\{j\delta^{[i]}, \eta^{[i]}\}$

Similarly, we have $\hat{w}^{[i]}(k+j-1|k) - w^{[i]}(k+j-1) \leq \hat{w}^{[i]}(k+j-1|k) - \max\{w^{[i]}(k-1)-j\delta^{[i]}, -\eta^{[i]}\} \leq \min\{j\delta^{[i]}, \eta^{[i]}\}$.

The above two inequalities suggest $\|w(k+j-1) - \hat{w}(k+j-1|k)\|_2 \leq \sqrt{\bar{\lambda}(P)}\|e(j)\|_2$, which further implies (7).

Second, we verify the result in (8). It can be observed that

$$\begin{aligned} Q &= |\hat{w}^{[i]}(k+j-1|k) - \hat{w}^{[i]}(k+j-1|k')| \\ &= \frac{1}{2}\Big| \min\{w^{[i]}(k-1)+j\delta^{[i]}, \eta^{[i]}\} \\ &\quad - \min\{j\delta^{[i]} - w^{[i]}(k-1), \eta^{[i]}\} \\ &\quad - \min\{w^{[i]}(k'-1) + (j+\Delta)\delta^{[i]}, \eta^{[i]}\} \\ &\quad + \min\{(j+\Delta)\delta^{[i]} - w^{[i]}(k'-1), \eta^{[i]}\}\Big|. \end{aligned} \tag{33}$$

Noting that $w^{[i]}(k'-1) + \Delta\delta^{[i]} \geq w^{[i]}(k-1)$ and $w^{[i]}(k'-1) - \Delta\delta^{[i]} \leq w^{[i]}(k-1)$, the value of Q can be analyzed by considering the following two groups of conditions, i.e.,

(a) $w^{[i]}(k'-1) + (j+\Delta)\delta^{[i]} \leq \eta^{[i]}$, $w^{[i]}(k-1) + j\delta^{[i]} \leq \eta^{[i]}$;
(b) $w^{[i]}(k'-1) + (j+\Delta)\delta^{[i]} > \eta^{[i]}$, $w^{[i]}(k-1) + j\delta^{[i]} > \eta^{[i]}$;
(c) $w^{[i]}(k'-1) + (j+\Delta)\delta^{[i]} > \eta^{[i]}$, $w^{[i]}(k-1) + j\delta^{[i]} \leq \eta^{[i]}$;
and
(A) $(j+\Delta)\delta^{[i]} - w^{[i]}(k'-1) < \eta^{[i]}$, $j\delta^{[i]} - w^{[i]}(k-1) < \eta^{[i]}$;
(B) $(j+\Delta)\delta^{[i]} - w^{[i]}(k'-1) \geq \eta^{[i]}$, $j\delta^{[i]} - w^{[i]}(k-1) \geq \eta^{[i]}$;
(C) $(j+\Delta)\delta^{[i]} - w^{[i]}(k'-1) \geq \eta^{[i]}$, $j\delta^{[i]} - w^{[i]}(k-1) < \eta^{[i]}$.

Combining the two groups of conditions yields nine cases. By discussing (33) for these cases, the result of Lemma 1 can be verified. Denote $e^{[i]}(\Delta) \triangleq \min\{\Delta\delta^{[i]}, \eta^{[i]}\}$.

1) If conditions (a) and (A) hold, then (33) can be rewritten as $Q = |w^{[i]}(k-1) - w^{[i]}(k'-1)| \leq \Delta\delta^{[i]}$. From conditions (a) and (A), we obtain $\Delta\delta^{[i]} \leq (j+\Delta)\delta^{[i]} \leq \eta^{[i]}$. That is, $|\hat{w}^{[i]}(k+j-1|k) - \hat{w}^{[i]}(k+j-1|k')| \leq e^{[i]}(\Delta)$.

2) If conditions (a) and (B) hold, then $Q = \frac{1}{2}|w^{[i]}(k-1) - w^{[i]}(k'-1) - \Delta\delta^{[i]}|$. On the one hand, $|w^{[i]}(k-1) - w^{[i]}(k'-1) - \Delta\delta^{[i]}| \leq 2\Delta\delta^{[i]}$; on the other hand, since $|w^{[i]}(k'-1) + \Delta\delta^{[i]}| \leq \eta^{[i]}$, we have $|w^{[i]}(k-1) - w^{[i]}(k'-1) - \Delta\delta^{[i]}| \leq 2\eta^{[i]}$. Hence, it holds that $|\hat{w}^{[i]}(k+j-1|k) - \hat{w}^{[i]}(k+j-1|k')| \leq e^{[i]}(\Delta)$.

3) If conditions (a) and (C) hold, then $Q = \frac{1}{2}|2w^{[i]}(k-1) - w^{[i]}(k'-1) - (j+\Delta)\delta^{[i]} + \eta^{[i]}|$. Let $R_3 = 2w^{[i]}(k-1) - w^{[i]}(k'-1) - (j+\Delta)\delta^{[i]} + \eta^{[i]}$. On the one hand, separately treating the two terms $w^{[i]}(k-1) - w^{[i]}(k'-1) - \Delta\delta^{[i]}$ and $w^{[i]}(k-1) - j\delta^{[i]} + \eta^{[i]}$ in R_3 yields $-2\Delta\delta^{[i]} \leq R_3 \leq 2\eta^{[i]}$. On the other hand, we also obtain $R_3 = 2w^{[i]}(k-1) - w^{[i]}(k'-1) - \Delta\delta^{[i]} - j\delta^{[i]} + \eta^{[i]} \leq w^{[i]}(k'-1) - j\delta^{[i]} + \eta^{[i]} + \Delta\delta^{[i]} \leq 2\Delta\delta^{[i]}$, and $R_3 = (2w^{[i]}(k-1) - j\delta^{[i]} + \eta^{[i]}) - w^{[i]}(k'-1) - \Delta\delta^{[i]} \geq j\delta^{[i]} - \eta^{[i]} - w^{[i]}(k'-1) - \Delta\delta^{[i]} \geq 2j\delta^{[i]} - 2\eta^{[i]} \geq -2\eta^{[i]}$. That is, $-2\eta^{[i]} \leq R_3 \leq 2\Delta\delta^{[i]}$. In summary, we obtain $Q \leq e^{[i]}(\Delta)$.

4) If conditions (b) and (A) hold, then $Q = \frac{1}{2}|w^{[i]}(k-1) - w^{[i]}(k'-1) + \Delta\delta^{[i]}|$. Similar to case 2), the inequality $Q \leq \Delta\delta^{[i]}$ can be easily shown. Moreover, we can also obtain $0 \leq w^{[i]}(k-1) - w^{[i]}(k'-1) + \Delta\delta^{[i]} \leq 2\eta^{[i]} - j\delta^{[i]} \leq 2\eta^{[i]}$, which implies $Q \leq \eta^{[i]}$.

5) If conditions (b) and (B) hold, then $Q = 0$.

6) If conditions (b) and (C) hold, then $Q = \frac{1}{2}|\eta^{[i]} - j\delta^{[i]} + w^{[i]}(k-1)|$. In fact, we directly have $\eta^{[i]} - j\delta^{[i]} + w^{[i]}(k-1) \geq 0$. Besides, both inequalities $\eta^{[i]} - j\delta^{[i]} + w^{[i]}(k-1) \leq 2\eta^{[i]} - j\delta^{[i]} \leq 2\eta^{[i]}$ and $\eta^{[i]} - j\delta^{[i]} + w^{[i]}(k-1) \leq -w^{[i]}(k'-1) + (j+\Delta)\delta^{[i]} - j\delta^{[i]} + w^{[i]}(k-1) \leq 2\Delta\delta^{[i]}$ hold, which verify $Q \leq e^{[i]}(\Delta)$.

7) If condition (c) and (A) hold, then $Q = \frac{1}{2}|2w^{[i]}(k-1) - w^{[i]}(k'-1) + (j+\Delta)\delta^{[i]} - \eta^{[i]}|$. Similar to case 3), we let $R_6 = 2w^{[i]}(k-1) - w^{[i]}(k'-1) + (j+\Delta)\delta^{[i]} - \eta^{[i]}$ and then using the conditions (c) and (A) we obtain $-2\eta^{[i]} \leq R_6 \leq 2\Delta\delta^{[i]}$. Besides, if we use the inequalities $w^{[i]}(k'-1) - \Delta\delta^{[i]} \leq w^{[i]}(k-1) \leq \eta^{[i]} - j\delta^{[i]}$,

one obtains $-2\Delta\delta^{[i]} \leq R_6 \leq 2\eta^{[i]}$. In conclusion, we have $Q \leq e^{[i]}(\Delta)$.

8) If condition (c) and (B) hold, then $Q = \frac{1}{2}|w^{[i]}(k-1) + j\delta^{[i]} - \eta^{[i]}|$. First, we have $w^{[i]}(k-1) + j\delta^{[i]} - \eta^{[i]} \leq 0$. By using $w^{[i]}(k-1) \geq w^{[i]}(k'-1) - \Delta\delta^{[i]}$ and $w^{[i]}(k'-1) + j\delta^{[i]} - \eta^{[i]} \geq -\Delta\delta^{[i]}$, we have $w^{[i]}(k-1) + j\delta^{[i]} - \eta^{[i]} \geq -2\Delta\delta^{[i]}$. We can also obtain $w^{[i]}(k-1) + j\delta^{[i]} - \eta^{[i]} \geq 2w^{[i]}(k-1) \geq -2\eta^{[i]}$. That is, it holds that $Q \leq e^{[i]}(\Delta)$.

9) If condition (c) and (C) hold, then $Q = |w^{[i]}(k-1)|$ Indeed, we have $Q \leq \eta^{[i]}$. Next, from (c) and (C), we obtain $j\delta^{[i]} - w^{[i]}(k-1) \leq \eta^{[i]} \leq w^{[i]}(k'-1) + (j+\Delta)\delta^{[i]}$, which implies $w^{[i]}(k-1) \geq -w^{[i]}(k'-1) - \Delta\delta^{[i]} \geq -w^{[i]}(k-1) - 2\Delta\delta^{[i]}$, that is, $w^{[i]}(k-1) > -\Delta\delta^{[i]}$. Besides, from inequality $w^{[i]}(k-1) + j\delta^{[i]} \leq \eta^{[i]} \leq (j+\Delta)\delta^{[i]} - w^{[i]}(k'-1)$, one has $w^{[i]}(k-1) \leq \Delta\delta^{[i]} - w^{[i]}(k'-1) \leq 2\Delta\delta^{[i]} - w^{[i]}(k-1)$, which implies $w^{[i]}(k-1) \leq \Delta\delta^{[i]}$. In summary, we obtain $Q \leq e^{[i]}(\Delta)$.

Incorporating the above nine cases, the inequality $Q \leq e^{[i]}(\Delta)$ holds, which further verifies the result (8).

References

[1] J. B. Rawlings, D. Q. Mayne, and M. Diehl, *Model Predictive Control: Theory, Computation, and Design*. Madison, WI, USA: Nob Hill, 2017.

[2] C. Liu, J. Gao, H. Li, and D. Xu, “Aperiodic robust model predictive control for constrained continuous-time nonlinear systems: An event-triggered approach,” *IEEE Trans. Cybern.*, vol. 48, no. 5, pp. 1397–1405, May 2018.

[3] K. Hashimoto, S. Adachi, and D. V. Dimarogonas, “Event-triggered intermittent sampling for nonlinear model predictive control,” *Automatica*, vol. 81, pp. 148–155, 2017.

[4] H. Li and Y. Shi, “Event-triggered robust model predictive control of continuous-time nonlinear systems,” *Automatica*, vol. 50, no. 5, pp. 1507–1513, 2014.

[5] C. Liu, H. Li, Y. Shi, and D. Xu, “Distributed event-triggered model predictive control of coupled nonlinear systems,” *SIAM J. Control Optim.*, vol. 58, no. 2, pp. 714–734, 2020.

[6] Z. Sun, Y. Xia, L. Dai, and P. Campoy, “Tracking of unicycle robots using event-based MPC with adaptive prediction horizon,” *IEEE/ASME Trans. Mechatronics*, vol. 25, no. 2, pp. 739–749, Apr. 2020.

[7] M. Wang, J. Sun, and J. Chen, “Input-to-state stability of perturbed nonlinear systems with event-triggered receding horizon control scheme,” *IEEE Trans. Ind. Electron.*, vol. 66, no. 8, pp. 6393–6403, Aug. 2019.

[8] Z. Sun, C. Li, J. Zhang, and Y. Xia, “Dynamic event-triggered MPC with shrinking prediction horizon and without terminal constraint,” *IEEE Trans. Cybern.*, to be published, doi: 10.1109/TCYB.2021.3081731.

[9] J. Yoo and K. H. Johansson, “Event-triggered model predictive control with a statistical learning,” *IEEE Trans. Syst., Man, Cyber., Syst.*, vol. 51, no. 4, pp. 2571–2581, Apr. 2021.

[10] Y. Zou, X. Su, S. Li, Y. Niu, and D. Li, “Event-triggered distributed predictive control for asynchronous coordination of multi-agent systems,” *Automatica*, vol. 99, pp. 92–98, 2019.

[11] N. He and D. Shi, “Event-based robust sampled-data model predictive control: A non-monotonic Lyapunov function approach,” *IEEE Trans. Circuits Syst. I*, vol. 62, no. 10, pp. 2555–2564, Oct. 2015.

[12] M. Rubagotti, A. Estrada, F. Castaños, A. Ferrara, and L. Fridman, “Integral sliding mode control for nonlinear systems with matched and unmatched perturbations,” *IEEE Trans. Autom. Control*, vol. 56, no. 11, pp. 2699–2704, Nov. 2011.

[13] Z. Sun, Y. Xia, L. Dai, K. Liu, and D. Ma, “Disturbance rejection MPC for tracking of wheeled mobile robot,” *IEEE/ASME Trans. Mechatronics*, vol. 22, no. 6, pp. 2576–2587, Dec. 2017.

[14] Y.-W. Liao and J. K. Hedrick, “Discrete-time integral sliding model predictive control for unmatched disturbance attenuation,” in *Proc. Amer. Control Conf.*, Boston, MA, USA, 2016, pp. 2675–2680.

[15] X. Lin and D. Görges, “Robust model predictive control of linear systems with predictable disturbance with application to multiobjective adaptive cruise control,” *IEEE Trans. Control Syst. Technol.*, vol. 28, no. 4, pp. 1460–1475, Jul. 2020.

[16] Z. Sun, L. Dai, K. Liu, D. V. Dimarogonas, and Y. Xia, “Robust self-triggered MPC with adaptive prediction horizon for perturbed nonlinear systems,” *IEEE Trans. Autom. Control*, vol. 64, no. 11, pp. 4780–4787, Nov. 2019.

[17] D. L. Marruedo, T. Alamo, and E. F. Camacho, “Input-to-state stable MPC for constrained discrete-time nonlinear systems with bounded additive uncertainties,” in *Proc. IEEE Conf. Decis. Control*, Las Vegas, NV, USA, 2002, pp. 4619–4624.

[18] D. M. Raimondo, D. Limon, M. Lazar, L. Magni, and E. F. Camacho, “Min-max model predictive control of nonlinear systems: A unifying overview on stability,” *Eur. J. Control*, vol. 15, no. 1, pp. 5–21, 2009.

[19] C. Liu, H. Li, J. Gao, and D. Xu, “Robust self-triggered min-max model predictive control for discrete-time nonlinear systems,” *Automatica*, vol. 89, pp. 333–339, 2018.

[20] D. E. Quevedo and D. Nesic, “Input-to-state stability of packetized predictive control over unreliable networks affected by packet-dropouts,” *IEEE Trans. Autom. Control*, vol. 56, no. 2, pp. 370–375, 2011.

[21] P. Li, Y. Kang, Y.-B. Zhao, and T. Wang, “A novel self-triggered MPC scheme for constrained input-affine nonlinear systems,” *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 68, no. 1, pp. 306–310, Jan. 2021.

[22] F. Castaños and L. Fridman, “Analysis and design of integral sliding manifolds for systems with unmatched perturbations,” *IEEE Trans. Autom. Control*, vol. 51, no. 5, pp. 853–858, May 2006.

[23] C. Rajhans, D. W. Griffith, S. C. Patwardhan, L. T. Biegler, and H. K. Pillai, “Terminal region characterization and stability analysis of discrete time quasi-infinite horizon nonlinear model predictive control,” *J. Process Control*, vol. 83, pp. 30–52, 2019.